PLASTIC DEFORMATION OF A MEDIUM UPON THE INTERACTION OF SHEAR SHOCK WAVES

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The problem of the interaction of shear shock waves in a plastically incompressible elastoplastic medium with reinforcement is discussed. Within the framework of the theory of small elasticoplastic strains, the mathematical model of the medium assumes their additivity: $e_{ij} = e^{e}_{ij} + e^{p}_{ij}$ (from left to right, respectively, the total, elastic, and plastic strains). The stress–strain state of the material is determined in the neighborhood of the interaction point, at which one can assume the fronts of the original waves Σ_1 and Σ_2 to be plane at a sufficiently great distance from the perturbation sources, forming an angle $0 < 2\alpha < \pi$ (see Fig. 1). The x_1, x_2, and x_3 axes are orthogonal. All the desired quantities are assumed to be independent of x_3; ahead of the fronts of the waves Σ_1 and Σ_2 the medium is in the free state: $\sigma_{ij}^{(0)} = e_{ij}^{(0)} = u_{ij}^{(0)} = 0$ (σ_{ij}, u_i are, respectively, the components of the stress and displacement tensors, i, j = 1, 2, 3). The numbers of the zones into which the space is divided by the wave fronts are denoted by the superscript in parentheses. The model of the medium postulates taking account of two reinforcement mechanisms [1]: kinematic and isotropic. Using the procedure of [2-4], first the elastic and then the elastoplastic self-similar solutions of the problem are constructed. In the process of interaction of the waves both dissipation-free regions of deformation of the material (elastic, neutral) and regions of plastic flow can be formed. In the dissipation-free regions the variation of the stresses and strains is determined by the elastic dependences, whereas in plastic regions one should make use of the equation of the loading surface and the associated law of plastic flow. We note that a similar problem of the interaction of irrotational shock waves in an elastoplastic space with reinforcement was solved in [5].

Without yet specifying the type of waves, let us consider the interaction of two shock fronts having the form of a step. This case is noteworthy in that it gives some ideas about the nature of the propagation of waves of a more general kind and an approximation for the initial instant of time which is necessary for solution of the general problem. It may turn out that the dissipation-free region fills the entire space as a result of interaction of the waves. In the coordinate system $x = x_1 - St$, $y = x_2$ the stress, velocity, and strain fields are then time-independent behind the fronts of the original waves, and one can assume the solution to be self-similar, i.e., one can assume that all the desired quantities depend only on $\xi \equiv \cot\varphi = xy^{-1}$, where φ is the angle measured from the positive x-axis counterclockwise (S is the velocity of a moving coordinate system tied to the interaction point of the waves). As follows from Fig. 1,

$$S = G(\sin\alpha)^{-1}, \tag{0.1}$$

where G is the propagation velocity of the original waves.

Using the linear law of Hooke and Cauchy's formulas and setting $u_1 = yu(\xi)$, $u_2 = yv(\xi)$, and $u_3 = yw(\xi)$, we obtain the following system of equations of motion (the prime denotes a derivative with respect to ξ, λ, and μ are the Lamé parameters, and ρ is the density of the medium):

$$\begin{gathered}(\lambda + 2\mu + \mu\xi^2 - \rho S^2)u'' - (\lambda + \mu)\xi v'' = 0,\\ (\lambda + \mu)\xi u'' - ((\lambda + 2\mu)\xi^2 + \mu - \rho S^2)v'' = 0, \quad (\mu + \mu\xi^2 - \rho S^2)w'' = 0.\end{gathered} \tag{0.2}$$

The solution of this system is trivial everywhere

$$u = a\xi + b,\ v = c\xi + d,\ w = l\xi + f, \tag{0.3}$$

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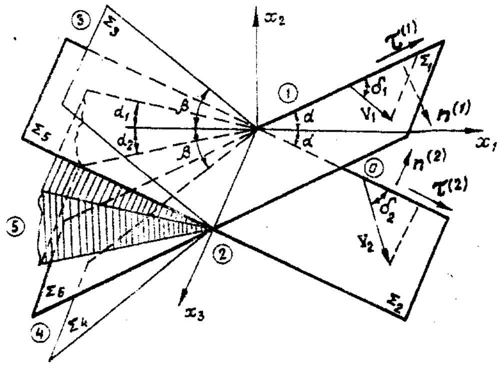

Fig. 1

here the determinant is different from zero (a, b, c, d, l, and f are constants). A nontrivial solution of the system (0.2) occurs upon the condition

$$(\rho G^2 - \mu)^2(\rho G^2 - (\lambda + 2\mu)) = 0, \quad (0.4)$$

where G is a new variable defined by the relationship

$$G^2(\xi^2 + 1) = S^2. \quad (0.5)$$

It follows from (0.4) that $G_1^2 = (\lambda + 2\mu)\rho^{-1}$, $G_{2,3}^2 = \mu\rho^{-1}$, i.e., both irrotational and shear shock waves can propagate in the body.

1. Elastic Solution. We shall discuss the case of the interaction of two shear shock waves propagating at an angle 2α with respect to each other. Then relationship (0.1) takes the form $S^2 \sin^2\alpha = \mu\rho^{-1}$. If one sets $G = G_1$, then we have $\varphi = \pm\beta + k\pi = \pm\arcsin[((\lambda + 2\mu)\mu^{-1})^{1/2}\sin\alpha] + k\pi$, from (0.5), which determines the position of the irrotational shock waves. If one sets $G = G_{2,3}$, then we have $\varphi = \pm\alpha + k\pi$ from (0.5) — the position of the shear shock waves. It follows from physical notions that k = 1. Since $|\sin\beta| \leqslant 1$, then $|(2(1-\nu)/(1-2\nu))^{1/2}\sin\alpha| \leqslant 1$, where ν is the Poisson coefficient, whence

$$0 < \alpha \leqslant \pi/4. \quad (1.1)$$

We shall denote waves in the upper half-plane y > 0 by odd subscripts Σ_1, Σ_3, and Σ_5, and in the lower half-plane y < 0 — by even subscripts Σ_2, Σ_4, and Σ_6. Here Σ_3 and Σ_4 are irrotational, and the rest are shear waves; Σ_6 and Σ_5 are a continuation, respectively, of Σ_1 and Σ_2 into the left half-plane x < 0; the numbers of the zones between the surfaces Σ_i (i = 1, 2, ..., 6) are indicated in Fig. 1.

However, we shall show that actually the waves Σ_3 and Σ_4 are absent in the solution determined by us.

Let the intensity of Σ_1 be equal to γ_1, and the intensity of Σ_2 be equal to γ_2; then from the consistency condition of Hadamard for the tangential displacement velocity v_τ we have (in the plane $x_3 = 0$)

$$[v_\tau]^{(0,1)} = -G[u_{\tau,n}]^{(0,1)} = G\gamma_1, \quad [v_\tau]^{(0,2)} = -G[u_{\tau,n}]^{(0,2)} = G\gamma_2,$$

where $G \equiv G_{2,3}$ is the propagation velocity of the original waves, n is the normal to the surface and [ ] is a discontinuity of the corresponding quantity. Along with γ_1 and γ_2 one should still specify $v_3^{(1)}$ and $v_3^{(2)}$ on Σ_1 and Σ_2. Then instead of γ_1, γ_2, $v_3^{(1)}$, $v_3^{(2)}$ one would specify the quantities V_1, V_2, δ_1, and δ_2, for example, by the formulas: $V_1\cos\delta_1 = -G\gamma_1$, $V_2\cos\delta_2 = -G\gamma_2$, $V_1\sin\delta_1 = v_3^{(1)}$, $V_2\sin\delta_2 = v_3^{(2)}$. With the fact that the components of the unit normal vector to the surface Σ_1 are equal to ($\sin\alpha$, $-\cos\alpha$) and those to the surface Σ_2 are equal to ($\sin\alpha$, $\cos\alpha$) taken into account, the displacement velocities in regions 1 and 2 are of the form

$$v_1^{(1)} = V_1\cos\delta_1\cos\alpha, \quad v_2^{(1)} = V_1\cos\delta_1\sin\alpha, \quad v_3^{(1)} = V_1\sin\delta_1,$$
$$v_1^{(2)} = V_2\cos\delta_2\cos\alpha, \quad v_2^{(2)} = -V_2\cos\delta_2\sin\alpha, \quad v_3^{(2)} = V_2\sin\delta_2.$$

Since in the moving coordinate system $v_j = -S\partial u_j/\partial x$ (j = 1, 2, 3), then setting

$$u_1^{(i)} = a_i x + b_i y, \quad u_2^{(i)} = c_i x + d_i y, \quad u_3^{(i)} = l_i x + f_i y, \quad (1.2)$$

on the basis of (0.3), we obtain (i = 1, 2)

$$a_i = \varkappa_i \sin\alpha,\ c_i = \omega_i \sin\alpha,\ l_i = (-1)^i \omega_i \tan\delta_i,$$
$$\varkappa_i = \gamma_i \cos\alpha,\ \omega_i = (-1)^{i-1}\gamma_i \sin\alpha. \quad (1.3)$$

We have

$$l_i = (-1)^i \varkappa_i \cos\alpha,\ d_i = (-1)^i \omega_i \cos\alpha,\ f_i = -\omega_i \tan\delta_i \cot\alpha. \quad (1.4)$$

from the condition of continuity of the displacements on Σ_1 and Σ_2. Using (1.3) and (1.4), one can obtain the strains from the Cauchy formulas and the stresses from Hooke's law in zones 1 and 2 (not summing over i)

$$\sigma_{11}^{(i)} = \mu\gamma_i \sin 2\alpha,\quad \sigma_{22}^{(i)} = -\mu\gamma_i \sin 2\alpha,\quad \sigma_{33}^{(i)} = 0,$$
$$\sigma_{12}^{(i)} = (-1)^i \mu\gamma_i \cos 2\alpha,\quad \sigma_{13}^{(i)} = \mu\gamma_i \sin\alpha\tan\delta_i,\quad \sigma_{23}^{(i)} = (-1)^i \mu\gamma_i \cos\alpha\tan\delta_i; \quad (1.5)$$

$$e_{11}^{(i)} = 0.5\gamma_i \sin 2\alpha,\quad e_{22}^{(i)} = -0.5\gamma_i \sin 2\alpha,\quad e_{33}^{(i)} = 0,$$
$$e_{12}^{(i)} = (-1)^i \gamma_i \cos 2\alpha,\quad e_{13}^{(i)} = 0.5\gamma_i \sin\alpha\tan\delta_i,\quad e_{23}^{(i)} = (-1)^i 0.5\gamma_i \cos\alpha\tan\delta_i. \quad (1.6)$$

Adopting for i = 3, 4, 5 a structure for writing the coefficients just as in (1.3) and (1.4), we obtain from the condition of continuity of the displacements on $\Sigma_3(\xi = \cot(\pi - \beta))$. $\Sigma_4(\xi = \cot(\pi + \beta))$, $\Sigma_5(\xi = \cot(\pi - \alpha))$, $\Sigma_6(\xi = \cot(\pi + \alpha))$ and the equating of the expressions for b_5, d_5, and f_5 obtained on the one hand upon passing through Σ_5 and on the other hand upon passing through Σ_6 into the fifth zone

$$2\varkappa_5 \cot\alpha = (\varkappa_3 + \varkappa_4)(\cot\alpha - \cot\beta) + (\varkappa_1 + \varkappa_2)(\cot\alpha + \cot\beta),$$
$$2\omega_5 \cot\alpha = (\omega_3 + \omega_4)(\cot\alpha - \cot\beta) + (\omega_1 + \omega_2)(\cot\alpha + \cot\beta).$$
$$2\omega_5 \tan\delta_5 \cot\alpha = (\omega_3 \tan\delta_3 - \omega_4 \tan\delta_4)(\cot\alpha - \cot\beta) + (\omega_1 \tan\delta_1 - \omega_2 \tan\delta_2)(\cot\alpha + \cot\beta). \quad (1.7)$$

It is natural to assume that the waves Σ_3 and Σ_4 cannot change the direction of the polarization of the motion of the medium) they only intensify or weaken its intensity; therefore we shall assume in the following that $\delta_2 = \delta_4$.

It is well known that on irrotational shock waves $[v_\tau] = 0$ (or $[_{\tau,n}] = 0$) and on shear waves $[v_n] = 0$ (or $[u_{n,n}] = 0$). At the same time $u_{\tau,n} = u_{k,j}\tau_k n_j$, and $u_{n,n} = u_{k,j} n_k n_j$, where τ_k are the components of the unit tangent vector to the surface of the wave. On Σ_3 we have: $\Sigma_3 : \tau_1^{(3)} = \cos\beta,\ \tau_2^{(3)} = -\sin\beta,\ n_1^{(3)} = \sin\beta,\ n_2^{(3)} = \cos\beta$; on $\Sigma_4 : \tau_1^{(4)} = \cos\beta,\ \tau_2^{(4)} = \sin\beta,\ n_1^{(4)} = \sin\beta,\ n_2^{(4)} = -\cos\beta$; on $\Sigma_5 : \tau_1^{(5)} = \cos\alpha,\ \tau_2^{(5)} = -\sin\alpha,\ n_1^{(5)} = \sin\alpha,\ n_2^{(5)} = \cos\alpha$; and on $\Sigma_6 : \tau_1^{(6)} = \cos\alpha,\ \tau_2^{(6)} = \sin\alpha,\ n_1^{(6)} = \sin\alpha,\ n_2^{(6)} = -\cos\alpha$. Then with (1.2)-(1.4) taken into account we obtain after transformations

$$(\varkappa_1 - \varkappa_3)\cot\beta = \omega_1 - \omega_3,\ (\varkappa_2 - \varkappa_4)\cot\beta = \omega_4 - \omega_2,\ (\varkappa_3 - \varkappa_5)\tan\alpha = \omega_5 - \omega_3,\ (\varkappa_4 - \varkappa_5)\tan\alpha = \omega_4 - \omega_5.$$

The system of equations (1.7) and (1.8) has the solution:

$$\varkappa_1 = \varkappa_3,\ \varkappa_2 = \varkappa_4,\ \varkappa_5 = \varkappa_1 + \varkappa_2,\ \omega_1 = \omega_3,\ \omega_2 = \omega_4,$$
$$\omega_5 = \omega_1 + \omega_2,\ \tan\delta_5 = (\omega_1 \tan\delta_1 - \omega_2 \tan\delta_2)(\omega_1 + \omega_2)^{-1}. \quad (1.9)$$

Determining the corresponding coefficients from (1.3) and (1.4), where now i = 3, 4, 5, one can obtain the components of the displacement vector, the strain tensor, and the stress tensor in the neighborhood of the wave interaction point. At the same time one can assume i = 1, 2, 3, 4 in the relationships (1.2), (1.5), and (1.6). Consequently, there are no surfaces Σ_3 and Σ_4 in the wave packet, and the constraint (1.1) of this section is removed. The solution in zone 5 is the superposition of the solutions in zones 1 and 2. We have for it

$$u_1^{(5)} = 0.5(\gamma_1 + \gamma_2)x\sin 2\alpha - (\gamma_1 + \gamma_2)y\cos^2\alpha,$$
$$u_2^{(5)} = (\gamma_1 - \gamma_2)x\sin^2\alpha + 0.5(\gamma_2 - \gamma_1)y\sin 2\alpha,\quad u_3^{(5)} = \gamma(x\sin\alpha - y\cos\alpha),\quad \gamma \equiv \gamma_1\tan\delta_1 + \gamma_2\tan\delta_2; \quad (1.10)$$

$$\sigma_{11}^{(5)} = ((\lambda + \mu)\gamma_2 + \mu\gamma_1)\sin 2\alpha,\quad \sigma_{22}^{(5)} = ((\lambda + \mu)\gamma_2 - \mu\gamma_1)\sin 2\alpha, \quad (1.11)$$
$$\sigma_{33}^{(5)} = \lambda\gamma_2 \sin 2\alpha,\quad \sigma_{12}^{(5)} = -\mu(\gamma_1\cos 2\alpha + \gamma_2),\quad \sigma_{13}^{(5)} = \gamma\mu\sin\alpha,\quad \sigma_{23}^{(5)} = -\gamma\mu\cos\alpha;$$

$$e_{11}^{(5)} = 0.5(\gamma_1 + \gamma_2)\sin 2\alpha, \quad e_{22}^{(5)} = 0.5(\gamma_2 - \gamma_1)\sin 2\alpha, \quad e_{33}^{(5)} = 0,$$
$$e_{12}^{(5)} = -0.5(\gamma_1 \cos 2\alpha + \gamma_2), \quad e_{13}^{(5)} = 0.5\gamma \sin\alpha, \quad e_{23}^{(5)} = -0.5\gamma\cos\alpha. \tag{1.12}$$

Thus, the elastic solution obtained for the problem completes the proof of our assertion on the behavior of the surfaces Σ_3 and Σ_4. In the following we will denote zones 1 and 3 by the number "1", zones 2 and 4 by the number "2", and zone 5 by the number "3".

2. Elastoplastic Solution. We shall discuss the case of motion of the medium plane-polarized along x_3 with the goal of obtaining an analytic solution of the problem. To this end we set $\delta_1 = \delta_2 = \pi/2$, whence $\gamma_1 = \gamma_2 = 0$, $V_1 = v_3^{(1)}$, $V_2 = v_3^{(2)}$, and the quantity γ is finite. Then only σ_{13}, σ_{23}, e_{13}^p, e_{23}^p, u_3, are different from zero; consequently, $S_{i3} = \sigma_{i3}$ $(i = 1, 2)$, where S_{i3} are the components of the stress deviator.

We shall assume that the solution obtained in Sec. 1 is valid in zones 1 and 2. Mathematically, we write this condition in the form

$$I_{(m)} = S_{i3}^{(m)} S_{i3}^{(m)} = z_m^2 k^2,$$

where m is the number of the zone, $0 < z_m \leqslant 1$, k is the yield stress in the case of pure shear, and I_m is a quantity which characterizes the intensity of the stresses. Dissipative regions can form in zone 3 only in the case in which the waves Σ_5 and Σ_6 become neutral [2, 3]. Surfaces of weak discontinuity α_1 and α_2 belonging to zone 3, on which the stresses, plastic strains, and displacement velocities are continuous and their first derivatives undergo discontinuity, should be the boundaries of these regions. Adopting this scheme for constructing the kinematics of the motion, we shall ascertain the condition under which it can be realized. Since $\gamma_1 \tan \delta_1 = -V_1 G^{-1}$ and $\gamma_2 \tan \delta_2 = -V_2 G^{-1}$, then using (1.5) and (1.11) we obtain

$$I_{(1)} = \mu\rho V_1^2 = z_1^2 k^2, \quad I_{(2)} = \mu\rho V_2^2 = z_2^2 k^2, \quad I_{(3)} = \mu\rho (V_1 + V_2)^2 = \left(\sqrt{I_{(1)}} + \sqrt{I_{(2)}}\right)^2. \tag{2.1}$$

respectively, for zones 1, 2, and 3. The material in the third zone can change into the plastic state upon the condition $I_{(3)} k^{-2} \geqslant 1$, whence

$$(z_1 + z_2)^2 \geqslant 1. \tag{2.2}$$

Let this inequality be satisfied. The plastic fan in the third zone should be located between the two neutral regions of this zone.

Starting the construction of the elastoplastic solution from the half-plane y > 0 in the counterclockwise sense if one looks from the positive direction of the x_3 axis, we determine the position of the loading wave $\varphi_1 = \pi - \alpha_1$ from the relationship

$$c_1 \sin\alpha = G \sin\alpha_1, \tag{2.3}$$

where c_1 is its propagation velocity, which is subject to determination. We shall find it from the following considerations. The basic system of equations which determines the continuous solution of the problem in the dissipative region of zone 3 has the form

$$\sigma_{i3,i} - \rho \dot{v}_3 = 0, \quad \dot{\sigma}_{i3} - \mu v_{3,i} + 2\mu \dot{e}_{i3}^p = 0,$$
$$(k + r\varkappa)\dot{e}_{i3}^p - (\sigma_{i3} - q e_{i3}^p)\dot{\varkappa} = 0, \quad (\sigma_{i3} - q e_{i3}^p)(\dot{\sigma}_{i3} - q\dot{e}_{i3}^p) - r(k + r\varkappa)\dot{\varkappa} = 0, \tag{2.4}$$

in the variables x_i and t, where $r \geqslant 0$, $q \geqslant 0$ are the reinforcement parameters of the material and $\varkappa = \int_0^t \sqrt{\dot{e}_{i3}^p \dot{e}_{i3}^p}\, dt$ is the Odquist parameter; the dot and comma denote differentiation with respect to the time and the coordinates, respectively. The shape of the loading surface is determined by multiplying by itself the third relationship of (2.4): $(\sigma_{i3} - q e_{i3}^p)(\sigma_{i3} - q e_{i3}^p) = (k + r\varkappa)^2$. The last relationship of (2.4) is obtained by differentiation of the loading surface with respect to the time.

Having written (2.4) at the discontinuities and applied the geometrical and kinematic first-order consistency conditions, we obtain similarly to [3]

$$c_1 = G\sqrt{1 - (\sigma_{i3}^{(3)} n_i)^2 k^{-2} (1 + a)^{-1}}, \tag{2.5}$$

where $n_1 = \sin\alpha_1$; $n_2 = \cos\alpha_1$; $a = (r+q)/2\mu \geqslant 0$; $\sigma_{i3}^{(3)}$ are the still-unknown stresses in the third zone on the surface α_1 and in front of it. We make use of the following relationship on the wave Σ_5 for the determination of $\sigma_{i_3}{}^{(3)}$:

$$-G[\sigma_{i3}]^{(1,3)} = \mu[v_3]^{(1,3)} n_i^{(5)}. \tag{2.6}$$

Finding V_1 from the first relationship of (2.1) and using (1.5), we obtain $\sigma_{13}^{(1)} = -z_1 k\sin\alpha$, and $\sigma_{23}^{(1)} = z_1 k\cos\alpha$. for the components of the stresses in the first zone. Substituting these values into (2.6), we determine the intensity of the wave Σ_5 from the condition $I_{(3)} = \sigma_{i3}^{(3)}\sigma_{i3}^{(3)} = k^2$

$$[v_3]^{(1,3)} = -kt_1(\sqrt{\mu\rho})^{-1}, \quad t_1 = z_1\cos 2\alpha \pm \sqrt{1 - z_1^2\sin^2 2\alpha}. \tag{2.7}$$

The "–" sign in front of the root is not appropriate, since for $z_1 = 1$ we obtain that Σ_5 is absent, which is impossible. Then we have from (2.6)

$$\sigma_{13}^{(3)} = -k(z_1 + t_1)\sin\alpha, \quad \sigma_{23}^{(3)} = k(z_1 - t_1)\cos\alpha. \tag{2.8}$$

In addition on the wave $\varphi = \varphi_1$ and in front of it

$$v_3^{(3)} = k(z_1 + t_1)(\sqrt{\mu\rho})^{-1}, \quad e_{i3}^{p(3)} = \varkappa^{(3)} = 0. \tag{2.9}$$

Thus substituting (2.8) into (2.5), we obtain from (2.3) a transcendental equation which determines the position of the wave α_1 for different values of z_1 and α:

$$z_1\cos(\alpha + \alpha_1) - t_1\cos(\alpha - \alpha_1) = 1 - \sin^2\alpha_1(\sin\alpha)^{-2}. \tag{2.10}$$

If the construction of the elastoplastic solution would begin from the half-plane $y < 0$ in the clockwise sense, then proceeding similarly to the preceding discussion, we would obtain the following relationships: $\sigma_{13}^{(2)} = -z_2 k\sin\alpha$, $\sigma_{23}^{(2)} = -z_2 k\cos\alpha$, instead of (2.7)

$$[v_3]^{(2,3)} = -kt_2(\sqrt{\mu\rho})^{-1}, \quad t_2 = z_2\cos 2\alpha \pm \sqrt{1 - z_2^2\sin^2 2\alpha}. \tag{2.11}$$

The "+" sign is chosen in the expression for t_2 for the very same reasons as in (2.7).

On the loading wave $\varphi_2 = \pi + \alpha_2$ and in front of it in the third zone (upon passing through Σ_6)

$$\sigma_{13}^{(3)} = -k(z_2 + t_2)\sin\alpha, \quad \sigma_{23}^{(3)} = k(t_2 - z_2)\cos\alpha; \tag{2.12}$$

$$v_3^{(3)} = k(z_2 + t_2)(\sqrt{\mu\rho})^{-1}, \quad e_{i3}^{p(3)} = \varkappa^{(3)} = 0. \tag{2.13}$$

would occur.

In order to obtain the solution in zone 3, it is necessary to integrate the system of equations (2.4), having written it in advance in the variable with the boundary conditions (2.8) and (2.9) on the wave φ_1. The system of ordinary differential equations will take the form

$$\begin{gathered}(\sigma_{13}' - \sigma_{23}'\operatorname{ctg}\varphi)\sin\alpha + \sqrt{\mu\rho}\,v_3' = 0, \quad \sigma_{13}' + \sqrt{\mu\rho}\,v_3'\sin\alpha + 2\mu e_{13}^{p\prime} = 0, \\ \sigma_{23}' - \sqrt{\mu\rho}\,v_3'\sin\alpha\cdot\operatorname{ctg}\varphi + 2\mu e_{23}^{p\prime} = 0, \quad Ke_{i3}^{p\prime} - \Sigma_{i3}\varkappa' = 0; \quad \Sigma_{i3}\Sigma_{i3}' - KK' = 0,\end{gathered} \tag{2.14}$$

where $\Sigma_{i3} = \sigma_{i3} - qe_{i3}^p$; and $K = k + r\varkappa$; $r\varkappa' = K'$ $(i = 1, 2)$. We have six equations with the six unknowns $\sigma_{i3}, e_{i3}^p, v_3, \varkappa$. These equations give the trivial solution

$$\sigma_{13}' = \sigma_{23}' = e_{13}^{p\prime} = e_{23}^{p\prime} = v_3' = \varkappa' = 0, \tag{2.15}$$

which determines the neutral state of the medium in zone 3. For this state we have the values (2.8) and (2.9) or (2.12) and (2.13) $(z_1 = z_2)$. A nontrivial (plastic) solution of the system (2.14) is possible upon the condition

$$K^2(a\sin^2\alpha(\sin\varphi)^{-2} - (1 + a)) + \sin^2\alpha(\Sigma_{13}\cot\varphi + \Sigma_{23})^2 = 0. \tag{2.16}$$

Satisfying the equation of the loading surface by the substitution

$$\Sigma_{13} = K\cos\psi, \ \Sigma_{23} = K\sin\psi, \tag{2.17}$$

we transform the relationship (2.16) to the form

$$\cos(\psi - \varphi) = ((\sin^2 \varphi/\sin^2 \alpha)(1 + a) - a)^{1/2} \equiv \eta(\varphi). \tag{2.18}$$

This relationship determines ψ as a function φ. We note that upon substitution of (2.17) into (2.14) the latter equation is satisfied identically.

Solving the remaining equations, we obtain

$$K = C \exp\left[b \int_{\varphi_1}^{\varphi} \psi' \operatorname{tg}(\psi - \varphi)\, d\varphi \right], \tag{2.19}$$

where $\psi' = 1 - \eta'(1 - \eta^2)^{-1/2}$; $b = r(r + q + 2\mu)^{-1}$; and C is an integration constant, which is determined from the condition of continuity of the value of (2.19) at $\varphi = \varphi_1$, whence C = k (we shall assume that $\psi - \varphi \neq \pm\pi/2$). Finally, (2.19) can be represented in the form

$$K = k(\eta_1/\eta)^b \exp\left[b \int_{\varphi_1}^{\varphi} \left(\frac{1 - \eta^2(\varphi)}{\eta^2(\varphi)} \right)^{1/2} d\varphi \right], \tag{2.20}$$

where $\eta_1 = \eta(\varphi_1)$.

The proof of the impossibility of the formation in a class of bounded solutions of a plastic shock wave on which $[e_{i3}^p] \neq 0$, can be carried out similarly to [3]; therefore we shall not dwell on it.

The condition that the energy dissipation rate be positive in the region which is plastically deformed, i.e., $\sigma_{i3}\dot{e}_{i3}^p > 0$, is equivalent to the two inequalities: $\sigma_{i3}e_{i3}^{p'} < 0$ for y > 0 and $\sigma_{i3}e_{i3}^{p'} > 0$ for y < 0. Each of these inequalities should te taken into account in connection with specific calculations. In the particular case of an ideally plastic medium the indicated inequalities change into the following: $\varkappa' < 0$ for y > 0 and $\varkappa' > 0$ for y < 0.

Using (2.20), one can obtain

$$\varkappa' = K(r + q + 2\mu)^{-1} \tan(\psi - \varphi)\cdot\psi'. \tag{2.21}$$

Then

$$e_{13}^p = \int_{\varphi_1}^{\varphi} \psi' \cos\psi d\varphi + C_{13},\ e_{23}^p = \int_{\varphi_1}^{\varphi} \psi' \sin\psi d\varphi + C_{23}, \tag{2.22}$$

follows from the fourth and fifth equations of (2.14) and $C_{13} = C_{23} = 0$, since $e_{13}^p = e_{23}^p = 0$ for $\varphi = \varphi_1$.

We obtain from (2.17) the stress components

$$\sigma_{13} = qe_{13}^p + K\cos\psi,\ \sigma_{23} = qe_{23}^p + K\sin\psi, \tag{2.23}$$

and from the first three equations of (2.14) an expression for the displacement velocity

$$v_3 = 2G \sin\alpha \int_{\varphi_1}^{\varphi} \frac{\sin\varphi \sin(\varphi - \psi)}{(\sin^2\varphi - \sin^2\alpha)} \varkappa' d\varphi + C_3. \tag{2.24}$$

The constant C_3 is determined from the condition that for $\varphi = \varphi_1$ the relationship (2.24) is equal to (2.9), whence

$$C_3 = k(z_1 + t_1)(\sqrt{\mu\rho})^{-1}. \tag{2.25}$$

We shall determine the position of the unloading wave φ_2 from the condition of continuity of the stresses for $\varphi = \varphi_2$. Thus equating (2.23) to (2.12), we have

$$\cos\psi = -(z_2 + t_2)\sin\alpha,\ \sin\psi = (t_2 - z_2)\cos\alpha. \tag{2.26}$$

with the second relationship of (2.13) taken into account. By adding here the relationship (2.18) in which φ has been substituted in place of φ_2, we obtain

$$\cos(\psi - \alpha_2) = \eta(\alpha_2), \tag{2.27}$$

and $\psi = \psi(\varphi_2)$ in (2.26) and (2.27). Multiplying the first equation of (2.26) by $\cos \alpha_2$ and the second one by $\sin \alpha_2$ and adding them, we obtain with (2.27) taken into account a transcendental equation for the determination of α_2 for different values of z_2 and α:

$$\eta(\alpha_2) + t_2 \sin(\alpha + \alpha_2) + z_2 \cos 2\alpha \sin(\alpha - \alpha_2) = 0. \tag{2.28}$$

We note that when calculating α_1 from (2.10) and α_2 from (2.28) one should take only those values which belong to the sector $\Sigma_5\Sigma_6$. Having determined the position of the wave α_2 from the formula (2.28), we find its propagation velocity c_2 from a relationship similar to (2.3). The equality of (2.24) with $\varphi = \varphi_2$ to the first expression of (2.13) serves as the criterion for the correctness of the numerical calculations for α_2. The problem has been solved.

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ELASTIC STRESSES NEAR JOINTS OF BOUNDARIES OF CRYSTALLITES SUBJECTED TO SELF-DISTORTIONS

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1. The strength and plasticity of solids depends to a large extent on their superatomic structure. For polycrystalline materials, these important structural elements include crystallites (grains), crystallite boundaries, and joints of crystallite boundaries (JCB). Recently, a number of investigators established that JCB (or joints of boundaries of fragments) can be locations for generation of microcracks both with active deformation [1, 2] and in the creep regime [3, 4]. The concentration of thermoelastic stresses near JCB often causes information of microscopic cracks in ceramic materials [5]. Elastic stresses, arising near JCB, play an important role in recrystallization processes [4] and superplastic deformation [6].

The concentration of elastic stresses near JCB could be a result of several factors: elastic inhomogeneity (or anisotropy) of the material, high-temperature slipping along crystallite boundaries and, finally, self-distortion of crystallites. Stresses near sharp elastic inhomogeneities were examined in [7]. The results in [8] permit estimating the elastic stresses related to slipping along intersecting crystallite boundaries. In this work, we examine the problem of finding the distribution of elastic stresses near JCB in the third case, when the joining crystallites undergo self-distortions. In this case, self-distortions are taken to mean any (plastic, thermal, magnetostrictive, etc.) distortions of crystallites of a nonelastic nature. It is convenient to calculate the stresses by methods of the continuum theory of dislocations and disclinations [9-11]. Internal elastic stresses can be represented as a superposition of fields of elastic stresses of distributed dislocations.

2. Let us examine n wedge-shaped crystallites with planar boundaries $OP^{(m)}$ ($m = 1, 2 \ldots n$), joining along the z axis of a Cartesian coordinate system x, y, z (Fig. 1). The z axis is perpendicular to the plane of the figure. We shall assume that the crystallites are infinite along the z axis and are subjected to homogeneous self distortion $\beta_{ik}^{(m)}$, where the index m corresponds to the number of the crystallite. In the general case, the distortions

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